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Accuracy of Approximations to the Navier-Stokes Equations

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Introduction

TO avoid the complexity and the large computational costs of solutions to the full Navier-Stokes equations, systems of truncated differential equations have been proposed[1-3] which offer advantages in computational efficiency while capturing all the physically relevant behavior. The hierarchy of these systems is: the classical boundary-layer equations with specified edge properties (usually the streamwise pressure distribution), the coupled boundary-layer/inviscid equations, the so-called thin-layer equations which discard streamwise diffusion, and the Navier-Stokes equations.

In the present Note, we consider each of these approximations applied to an incompressible, laminar-separating flow at low and moderate Reynolds numbers.

Analysis

If we restrict our attention to two-dimensional, incompressible steady laminar flow, the entire hierarchy can be represented by the nondimensional equations

$$\omega = \frac{\partial^2\psi}{\partial y^2} + \frac{K_1}{R_E}\frac{\partial^2\psi}{\partial x^2}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} = \frac{\partial^2\omega}{\partial y^2} + \frac{K_2}{R_E}\frac{\partial^2\omega}{\partial x^2} \qquad (1)$$

where ψ is the dimensionless stream function and ω is the dimensionless vorticity. (For definitions of the transformation variables see Ref. 4.)

A particular member of the above hierarchy can be obtained by the appropriate choice of K_1 and K_2, as shown in Table 1.

Using a slightly modified version of the computer code of Ref. 4 applied to Eq. (1) we can treat each of these levels of approximation *within a single logical and numerical framework so that differences between solutions can be attributed to the equations solved and not to the method of solution.* We apply this code to a strongly retarded flow, at two Reynolds numbers, $UL/\nu = 9{,}000$ and 900,000. For discussion purposes, approximations (2) and (3) in Table 1 will be termed higher approximations.

Received Aug. 2, 1982; revision received Feb. 16, 1983.

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Results and Discussion

In the present case, the low Reynolds number problem for the strongly retarded flow corresponds to Briley's case 4,[5] the high Reynolds number case has the same boundary conditions but the viscosity has been reduced by a factor of 10^2. Comparisons of the results obtained using the methods of Refs. 4 and 5 are considered in Ref. 4. Figure 1 shows the freestream velocity distribution, imposed at the outer computational boundary for this case.

Figure 2 shows the nondimensional skin friction vs distance along the plate for a strongly retarded flow. The higher approximations do very well and the principal higher-order effect is the interaction between the viscous and inviscid flows. Each of the higher approximations can compute through the separation point without difficulty. Although the coupled boundary-layer/inviscid calculation differs most from the full equations, it is still quite satisfactory despite a very low Reynolds number.

To determine the effect of Reynolds number, the same case was rerun at a Reynolds number 100 times larger. In Fig. 3, the results for the strongly retarded flow at high Reynolds number are presented, and again the higher approximations are indistinguishable from the solutions to the full equations.

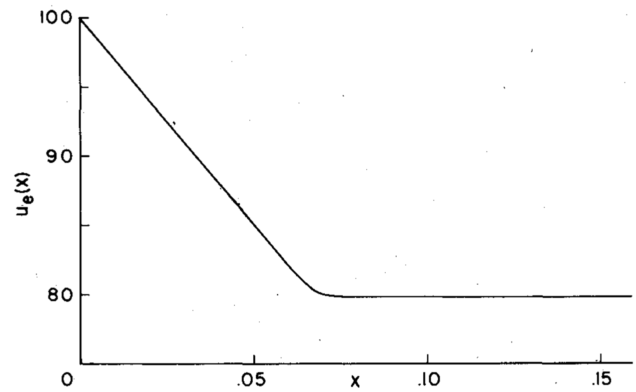

Fig. 1 Freestream velocity distribution.

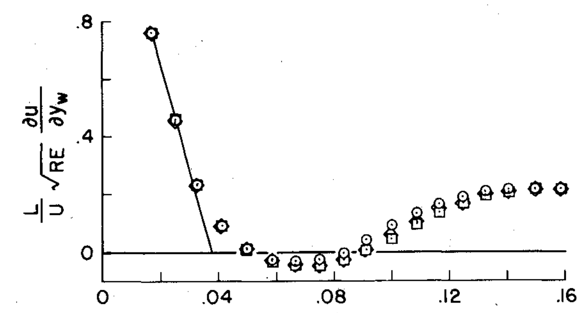

Fig. 2 Comparison of dimensionless wall shear stress in low Reynolds number strongly retarded flow.

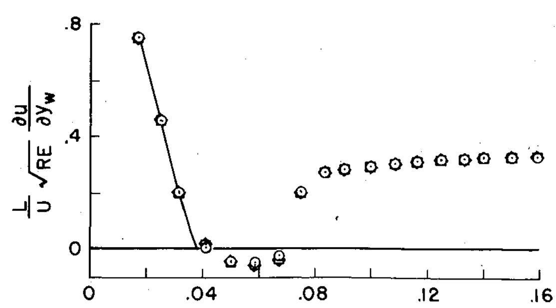

Fig. 3 Comparison of dimensionless wall shear stress in a high Reynolds number strongly retarded flow.

Table 1 Constant values in Eq. (1)

Equation	K_1	K_2	Region
(1) Boundary layer	0	0	For all Y
(2) Coupled boundary	0	0	$Y \leq \delta$
layer/inviscid[a]	1	0	$Y > \delta$
(3) Thin layer	1	0	For all Y
(4) Navier-Stokes	1	1	For all Y

[a]Also requires $\omega = 0$ for $Y > \delta$.

While conclusions that may be drawn on the basis of these solutions are strictly valid only for incompressible laminar flow, they can be plausibly assumed to apply to a much larger class of flows. We note that the differences between the various approximations and the full equations decrease with increasing Reynolds number as expected. Since most aerodynamically significant flows are at high Reynolds numbers, the error introduced by the approximation to the governing equations will tend to be small and will certainly be insignificant relative to the errors introduced by the turbulence model. Compressibility effects should not alter the present conclusions insofar as viscous effects are concerned, and the direct effects of shock waves and normal pressure gradients within the boundary layer as sources of error should be relatively small for moderate Mach numbers. In this regard, the recent comparisons of Navier-Stokes and thin-layer solutions for supersonic corner flow[6] are informative.

For any flow, or region of flow, for which viscous-inviscid interaction effects are small, classical boundary-layer equations will provide a satisfactory description of the viscous flow at a fraction of the computational cost of any higher approximation. For flows with significant viscous-inviscid interaction, as long as they are boundary-layer-like, i.e., $v/u \ll 1$, coupled boundary-layer/inviscid equations will provide an adequate flow description.

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Decomposition of a Disturbance in Parallel Shear Flows

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IT is well known[1] that, *locally* in space and time, a small perturbation in a fluid may be decomposed into three types of nearly independent fluctuations: vorticity, pressure, and entropy modes. Roughly speaking, both the vorticity and entropy modes convect with the fluid while inducing solenoidal and irrotational velocity fields, respectively, without significant pressure fluctuations. Indeed, most of the pressure fluctuations are carried by acoustic waves that propagate relative to the fluid at the speed of sound. The velocity field associated with the pressure fluctuations is irrotational. Because of the nonlinearity of the equations of motion, these disturbances interact quadratically whenever their amplitudes are no longer infinitesimal. For example, vorticity-vorticity interactions are responsible for the generation of sound by turbulence whereas sound-sound interactions lead to shock waves in a finite amplitude pressure field.

Received Jan. 28, 1983.

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While a *local* study of the flow is extremely important and useful, such study cannot provide much information on the *global* behavior of a disturbance except when the disturbance is propagating through a uniform undisturbed state. The reason is that the interaction between the mean flow gradients and the disturbance is ignored.

An important extension of these classical ideas on the decomposition of a flow is given by Goldstein.[2] Goldstein shows that for irrotational base flows, the perturbation velocity $\boldsymbol{u}$ may be decomposed into

$$\boldsymbol{u} = \nabla G + \boldsymbol{v} \tag{1a}$$

where, for incompressible flows,

$$\Delta G = -\nabla \cdot \boldsymbol{v} \tag{1b}$$

Δ is the Laplacian and G the velocity potential. The velocity field $\boldsymbol{v}$, which is called a *generalized convected disturbance*, satisfies

$$\frac{D\boldsymbol{v}}{Dt} + \boldsymbol{v} \cdot \nabla U = 0 \tag{1c}$$

where D/Dt denotes the time derivative following the irrotational mean flow velocity U, and the perturbation pressure p is given by

$$\frac{p}{\rho} = -\frac{DG}{Dt} \tag{1d}$$

where $\rho = $ const is the fluid density. Note that Eqs. (1) are valid for constant density flows without entropy fluctuations. Quite generally, Eq. (1c) can be solved in closed form[2] and the complete perturbation velocity field $\boldsymbol{u}$ may be obtained by solving a single Poisson equation (1b). Perhaps it is worth pointing out that Goldstein's results[2] are valid also for compressible flows and, in that case, significant simplifications may be obtained by making the tangent gas approximation.[3]

It is not possible to extend these ideas in a perfectly satisfactory way to rotational base flows. However, for parallel shear flows, Goldstein[4] has given a valid decomposition. The purpose of this Note is to show that the decomposition is not unique, and an alternate one which resembles Eqs. (1) quite closely can be derived from elementary considerations.

Consider a right-handed Cartesian coordinate system $\boldsymbol{x} = (x, y, z)$, fixed in space, such that the x axis is along the mean flow direction. A *simple* incompressible parallel shear flow is defined by the conditions that the unperturbed pressure and density are uniform and the velocity has components $[U(y), 0, 0]$. It is easy to see that such flow satisfies the equations of motion.

If this flow is perturbed by a disturbance, the linearized equations describing the evolution of the disturbance are

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2a}$$